[3]Tam, C.K.W., "Discrete Tones of Isolated Airfoils," *Journal of the Acoustical Society of America*, Vol. 55, June 1974, pp. 1173-1177.

[4]Rossiter, J.E., "Wind-Tunnel Experiments on the Flow Over Rectangular Cavities at Subsonic and Transonic Speeds," RAE Tech., Rept. 64037, 1964, and Reports and Memoranda, No. 3438, Oct. 1964.

[5]Ho, C.M. and Nosseir, N.S., "Dynamics of an Impinging Jet. Part. 1. The Feedback Phenomenon," *Journal of Fluid Mechanics*, Vol. 105, 1981, pp. 119-142.

Mach Reflection and Aerodynamic Choking in Two-Dimensional Ducted Flow

Ajay Kumar*
NASA Langley Research Center, Hampton, Virginia

Introduction

It is well known in the quasi-one-dimensional (1-D) theory for flow through a converging-diverging duct that, for a given throat to inflow area ratio A_T/A_i, there exists a supersonic inflow Mach number M_i below which the duct will choke aerodynamically (see, for example, Ref. 1). This Mach number M_i, within the assumptions of quasi-1-D flow, does not depend on how the change in inflow area to throat area is distributed in the axial direction. Therefore, it is possible to construct a single curve of A_T/A_i as a function of the smallest value of supersonic inflow Mach number M_i, which provides sonic throat conditions for quasi-1-D flow. This curve can then be used to determine whether a duct with a certain area ratio will choke or not for a given inflow Mach number. (This Note considers only the supersonic inflow conditions although a similar curve can be constructed for the subsonic inflow conditions as well.) In the present study, flow through a two-dimensional (2-D) duct with supersonic inflow is investigated numerically from the point of view of formation of Mach reflection, aerodynamic choking, and the possibility of constructing a curve similar to that for the quasi-1-D flow discussed above.

Results and Discussion

The Mach numbers and area ratios for which the calculations are made are tabulated in Table 1. The area ratio is varied by moving the lower boundary, AB, up or down. A grid of 55×61 points is used in the calculations. Results of these calculations are shown in Figs. 3-5.

Figure 3 shows the pressure contours in the first configuration at various inflow Mach numbers and at an area ratio of 0.7085. It is seen from this figure that at Mach 2.2 the shock wave undergoes regular or simple reflections, but as the inflow Mach number is decreased to 2.1 formation of a Mach reflection takes place at the lower boundary. This kind of reflection occurs when the Mach number behind the incident shock is lower than the detachment Mach number for the angle of flow turning on the incident boundary. Under such a situation, no simple shock reflection is possible thus causing a portion of the flow to pass through a normal or nearly normal shock that appears near the incident boundary. It also produces a small region of subsonic flow behind the normal shock. As the Mach number is decreased further to 2.0 and 1.95, the extent of Mach reflection is seen to grow in size thus producing a larger and larger region of subsonic flow. At Mach 1.9, the duct flow chokes causing a normal shock to stand slightly upstream of the duct entrance. Under the choked conditions, a portion of the flow approaching the duct is spilled around the leading edge of the duct upper boundary and the flow in the duct is again established with subsonic inflow and sonic throat.

Figure 4 shows the mass flux contours at Mach 1.95 and 1.9. The spilling of the flow at Mach 1.9 can be seen clearly from this figure. Also notice from Figs. 3 and 4 that no interaction between the flow through the duct and the flow in the extended region takes place until the inflow Mach number is reduced so as to choke the duct. Similar calculations were also made at area ratios of 0.638 and 0.7989 for the first configuration. Based on these calculations, Fig. 5 shows line plots of the inflow Mach numbers at which the duct chokes and the duct flow starts undergoing simple shock reflections against the area ratio. Also shown in this figure is a plot of the inflow Mach number at which the quasi-1-D flow chokes. It is seen that the 2-D flow chokes at a higher Mach number than the quasi-1-D flow for a given area ratio. Further, there is a distinct Mach number range above the choking Mach number in which the 2-D flow undergoes a Mach reflection. For example, at the area ratio of 0.7085, the quasi-1-D flow chokes at Mach 1.775 and the 2-D flow chokes around Mach 1.9. Between Mach 1.9 and 2.2, the 2-D flow undergoes a Mach reflection. In contrast, calculations for the second configuration at the same area ratio showed that choking occurred around Mach 1.7925 and simple shock reflections started taking place at Mach 1.825. Based on the preceding calculations for the two 2-D ducts, the following conclusions can be drawn.

Analysis

In the present investigation, two 2-D configurations, shown in Fig. 1, are analyzed. The first configuration has a large in-

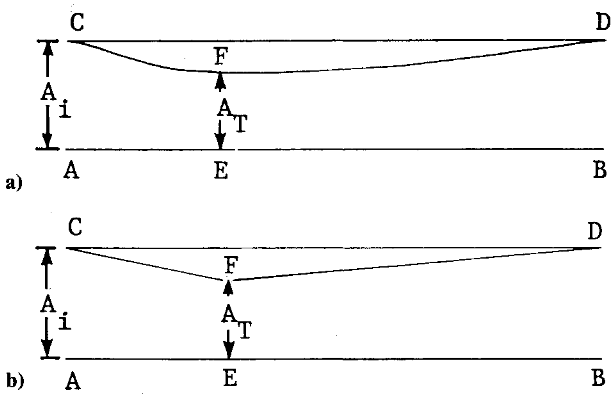

Fig. 1 a) Geometry of the first and b) second configurations.

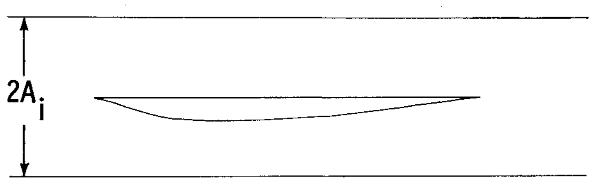

Fig. 2 Physical domain of computation.

Table 1 Area ratios and inflow Mach numbers

	Inflow Mach numbers for	
A_T/A_i	First configuration	Second configuration
0.638	2.05, 2.1, 2.15, 2.25, 2.3	—
0.7085	1.9, 1.95, 2.0, 2.1, 2.2	1.7925, 1.8, 1.825
0.7985	1.75, 1.8, 1.9, 2.0, 2.05	—

Received July 18, 1985; revision received Aug. 19, 1985.

*Aerospace Engineer, Computational Methods Branch, High-Speed Aerodynamics Division. Member AIAA.

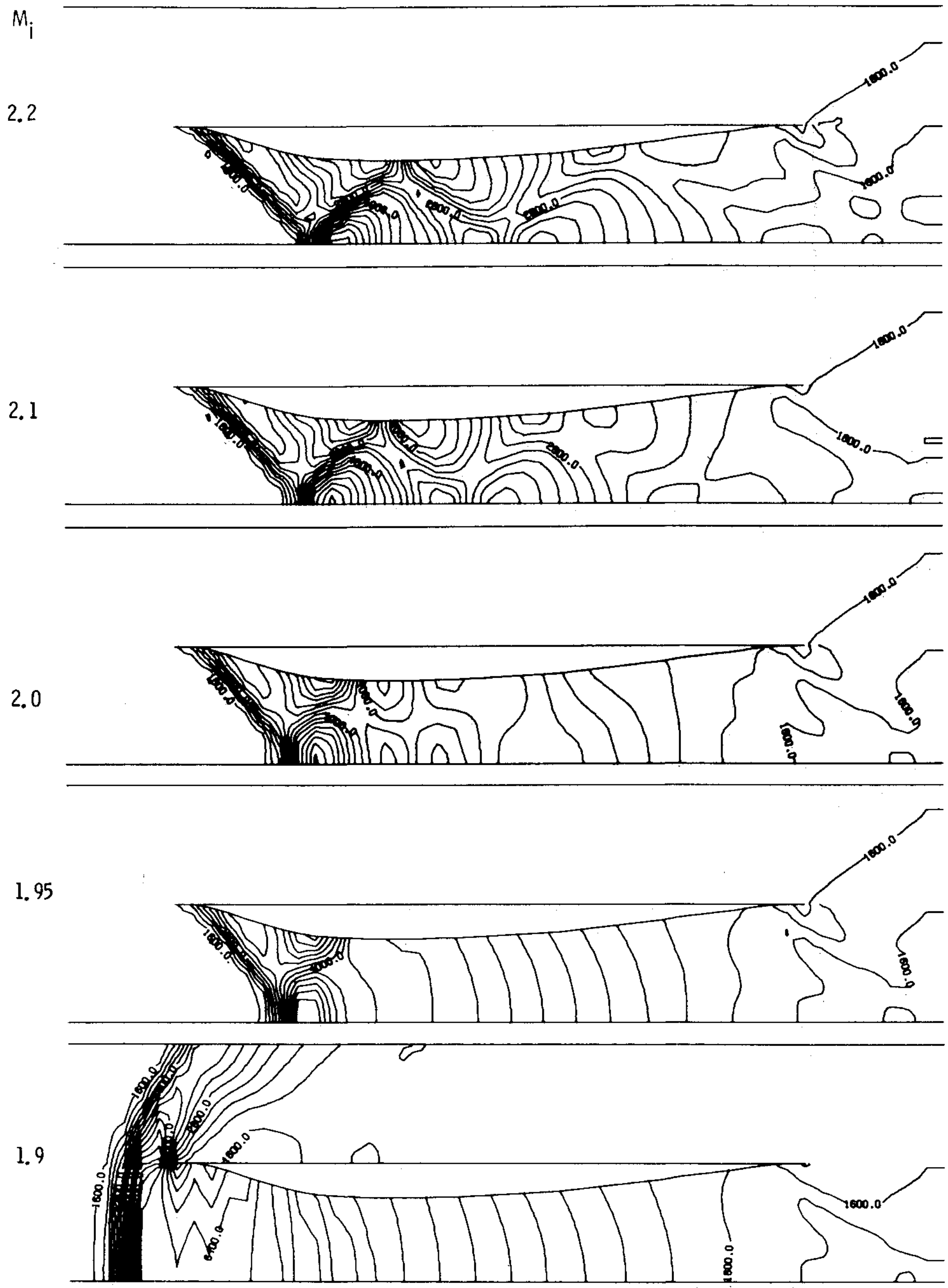

Fig. 3 Pressure contours in the first configuration at $A_T/A_i = 0.7085$.

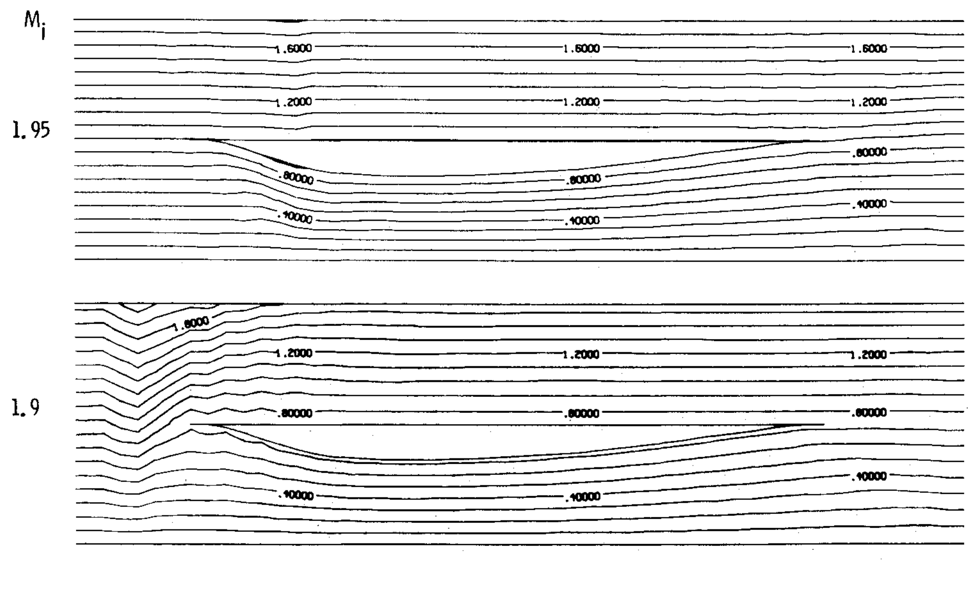

Fig. 4 Mass flux contours in the first configuration at $A_T/A_i = 0.7085$.

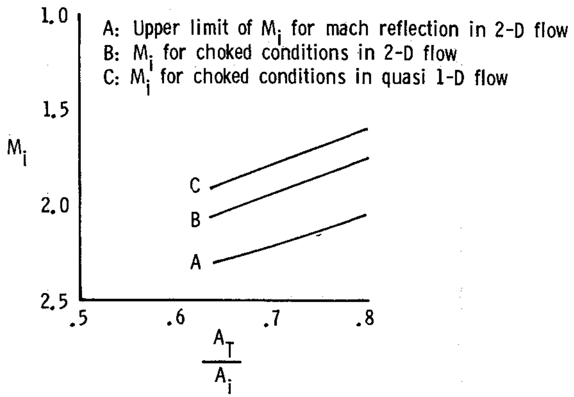

Fig. 5 Area ratio vs inflow Mach number for the first configuration.

itial compression (about 19) deg over a short distance and then a continuous expansion around the throat. This particular configuration is chosen to produce a sufficiently strong shock, thus causing significant total pressure losses. The second configuration has a 9.42-deg compression wedge followed by a 14.43-deg expansion at the throat. These angles are such that, for a given inflow area A_i, both configurations have the same throat area. Further, the two configurations have the same length and same axial location of the throat. The initial compression angle in the second configuration produces a shock through which the losses are much less as compared to the first configuration. In order to be able to numerically predict aerodynamic choking, if it occurs, both configurations are embedded in a freestream, as shown in Fig. 2. This extension of the physical domain of computation will allow a normal shock to stand in front of the duct in case of aerodynamic choking, thus providing a mechanism to spill some flow around the compression ramp. Under these conditions, the predicted flow through the duct will be physically meaningful with subsonic inflow and sonic throat conditions. However, if the inflow Mach number is high enough so as not to cause choking of the duct, the flow in the extended region should not have any influence on the duct flow.

The flow through these configurations is calculated using a two-dimensional code described in Ref. 2, which solves the Euler equations in full-conservation form by the well-known method of MacCormack.[3] In order to facilitate the use of general geometry with embedded bodies, an algebraic coordinate transformation is used in the code to generate a set of boundary-fitted curvilinear coordinates. The code as such is operational on the VPS 32 computer system at the NASA Langley Research Center and has been optimized to take maximum advantage of the vector processing capability of the system. A user's guide for the code is given in Ref. 4.

1) The 2-D flow always chokes at a higher Mach number than the corresponding quasi-1-D flow for a given throat to inflow area ratio. This is a consequence of the losses involved in the 2-D compression through oblique shocks. For quasi-1-D flow, the compression through the converging duct is isentropic. The difference in the choking Mach numbers between the 2-D and quasi-1-D flow depends upon the strength of the shock, or, in other words, upon the losses due to the entropy gains in the 2-D flow. For the first configuration, where the initial shock is quite strong, the choking Mach numbers at an area ratio of 0.7085 differ significantly (1.9 for 2-D to 1.775 for quasi-1-D), whereas for the second configuration, where the initial shock is relatively weak, the choking Mach numbers differ only by a small amount (1.7925 for 2-D to 1.775 for quasi-1-D).

2) It is possible to construct a curve similar to that for the quasi-1-D flow of area ratio A_T/A_i against the inflow Mach number M_i at which the 2-D flow chokes. However, unlike quasi-1-D flow, where this curve depends only on the area ratio value, for the 2-D flow it also depends on the actual duct geometry, or, in other words, on how the area change from A_i to A_T is distributed in the axial direction. Thus, the curve will change as the actual geometry changes.

3) There is a Mach number range above the choking Mach number in which Mach reflection occurs in the 2-D flow. This range depends on the actual geometry. For Mach numbers above this range, simple shock reflections occur.

Acknowledgment

The author would like to thank M.D. Salas of NASA Langley Research Center for many useful discussions regarding this work.

References

[1]Liepmann, H.W. and Roshko, A., *Elements of Gasdynamics*, John Wiley & Sons, New York, 1966, p. 126.

[2]Kumar, A. "Numerical Analysis of the Scramjet Inlet Flow Field Using Two-Dimensional Navier-Stokes Equations," NASA TP-1940, Dec. 1981.

[3]MacCormack, R.W., "The Effect of Viscosity in Hypervelocity Impact Cratering," AIAA Paper 69-354, May 1969.

[4]Kumar, A., "User's Guide for NASCRIN—A Vectorized Code for Calculating Two-Dimensional Supersonic Internal Flow Fields," NASA TM-85708, Feb. 1984.